

# 2021 Virtual School on Electron-Phonon Physics and the EPW code

June 14-18 2021



U.S. DEPARTMENT OF  
**ENERGY**

**TACC**

Lecture Thu.2

# Polarons from first principles

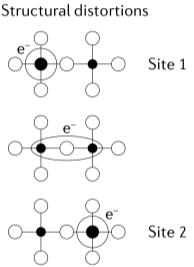
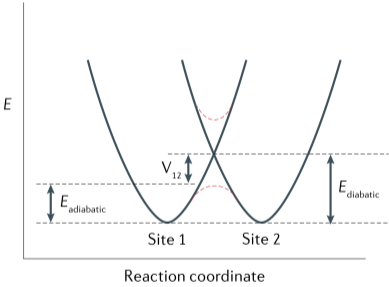
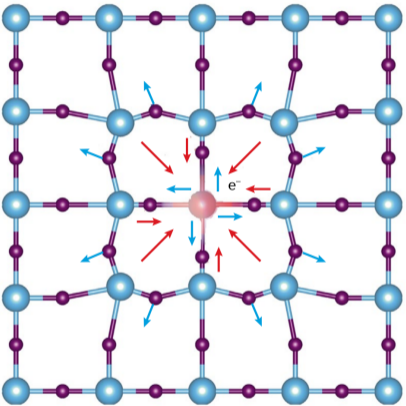
Feliciano Giustino

Oden Institute & Department of Physics

The University of Texas at Austin

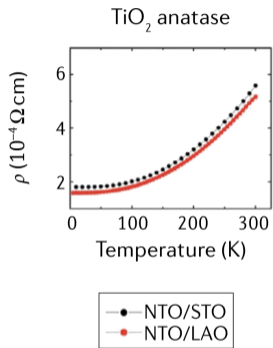
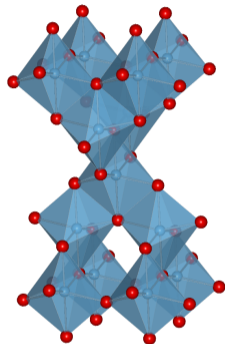
- Introduction to the polaron concept
- Photoemission signatures of polarons
- Many-body calculations of polaron satellites
- DFT calculations of polarons
- *Ab initio* polaron equations
- Open questions in polaron physics

# Intuitive notion of polaron

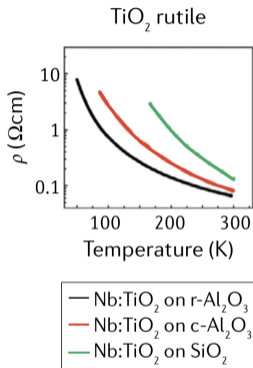


Figures from Franchini et al, Nat. Rev. Mater. 2021, 10.1038/s41578-021-00289-w

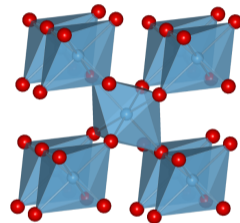
# Transport signatures of polarons



Diffusive



Activated



Hall mobility data from Zhang et al, J. Appl. Phys. 102, 013701 (2007);  
see discussion in Franchini et al, Nat. Rev. Mater. 2021, 10.1038/s41578-021-00289-w

# Angle-resolved photoelectron spectroscopy (ARPES)

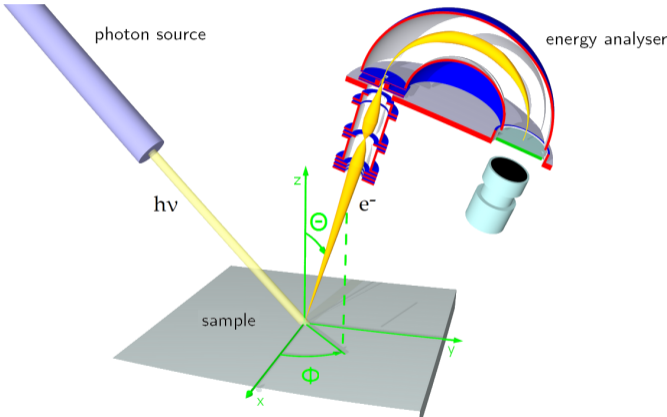
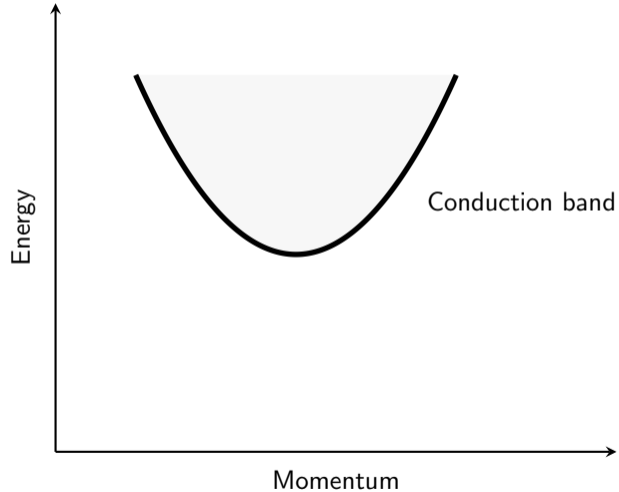
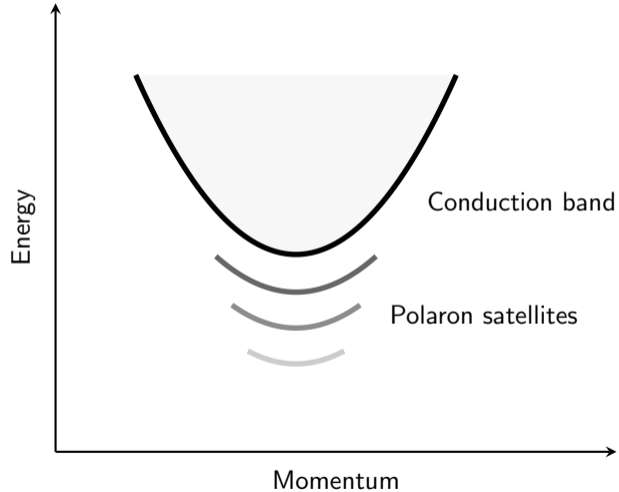


Figure from commons.wikimedia.org/wiki/File:ARPESgeneral.png

# Polaron satellites



# Polaron satellites





# Polaron satellites in anatase $\text{TiO}_2$

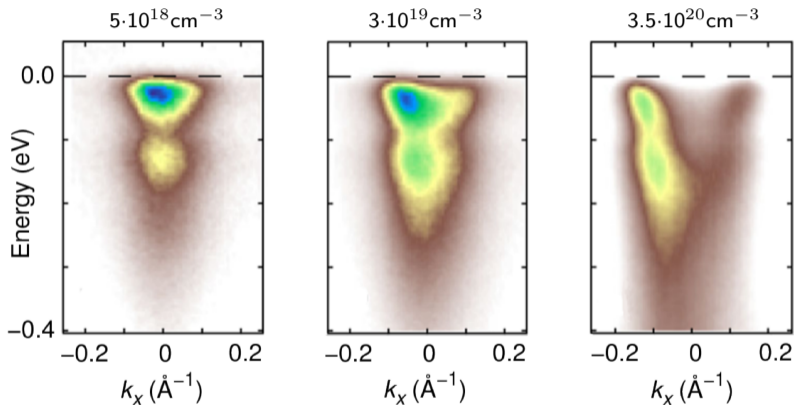


Figure from Moser et al, Phys. Rev. Lett. 110, 196403 (2013)

# Polaron satellites at the SrTiO<sub>3</sub>(001) surface

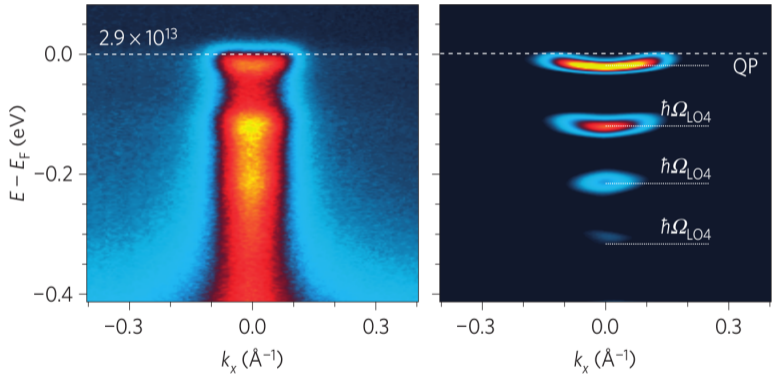


Figure from Wang et al, Nature Mater. 15, 835 (2016)

# Polaron satellites in EuO

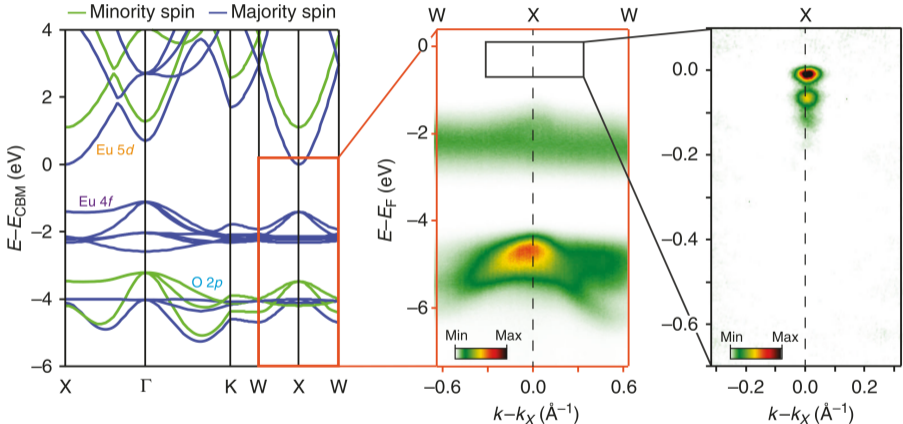


Figure from Riley et al, Nat. Commun. 9, 2305 (2018)

# Kinks vs. satellites in ARPES

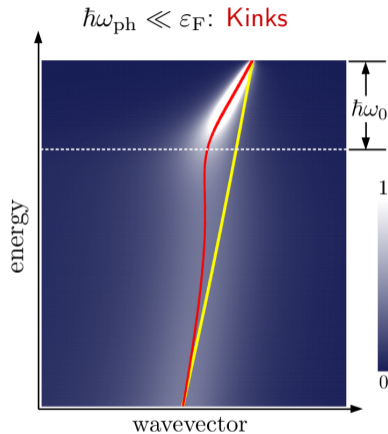


Figure from FG, Rev. Mod. Phys. 89, 015003 (2017)

# Kinks vs. satellites in ARPES

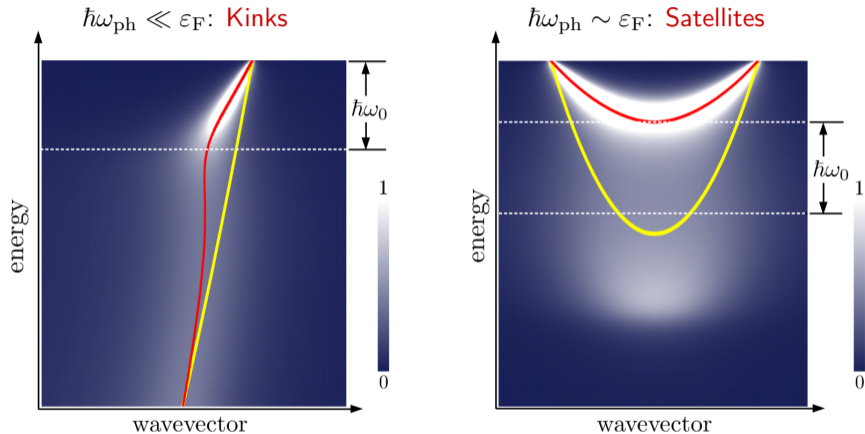


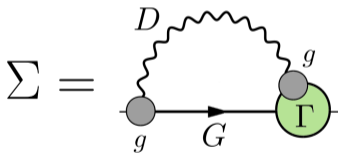
Figure from FG, Rev. Mod. Phys. 89, 015003 (2017)

Quasiparticle equation (from Lecture Tue.2)

$$\left[ -\frac{\hbar^2}{2m_e} \nabla^2 + V_{\text{tot}}(\mathbf{r}) \right] f_s(\mathbf{x}) + \int d\mathbf{x}' \underline{\Sigma(\mathbf{x}, \mathbf{x}', \varepsilon_s/\hbar)} f_s(\mathbf{x}') = \varepsilon_s f_s(\mathbf{x})$$

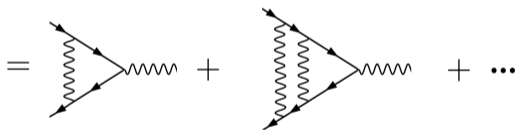
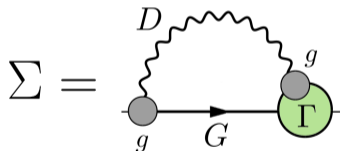
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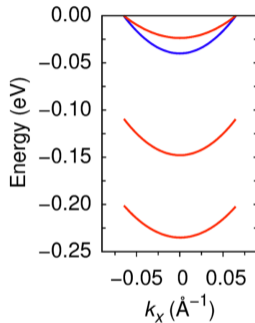
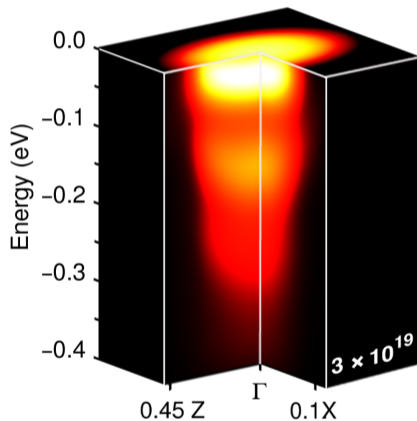
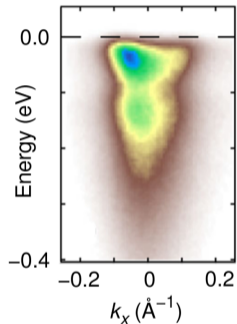
Cumulant expansion method

Aryasetiawan et al, Phys. Rev. Lett. 77, 2268 (1996); Zhou et al, J. Chem. Phys. 143, 184109 (2015);  
 Gumhalter et al, Phys. Rev. B 94, 035103 (2016); Nery et al, Phys. Rev. B 97 (2018)



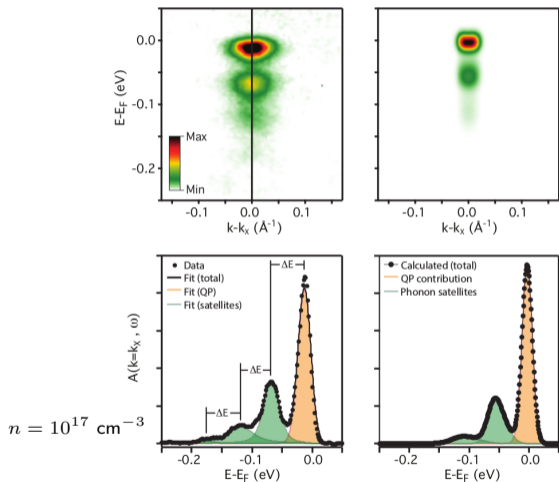
# Calculated vs. measured spectral function: $\text{TiO}_2$

Moser et al,  
PRL 110, 196403 (2013)



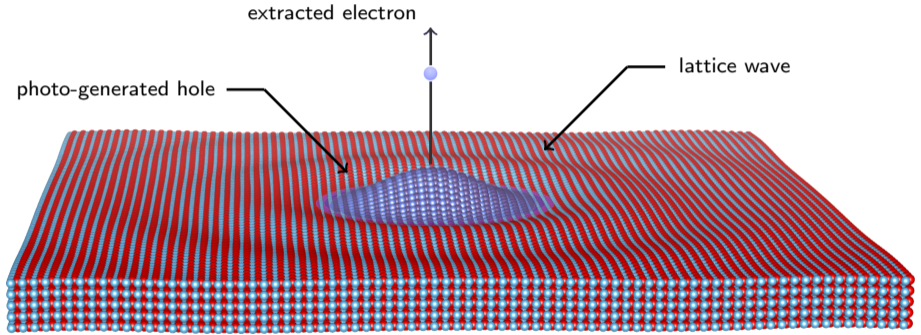
Verdi et al, Nat. Commun. 8, 15769 (2017)

# Calculated vs. measured spectral function: EuO

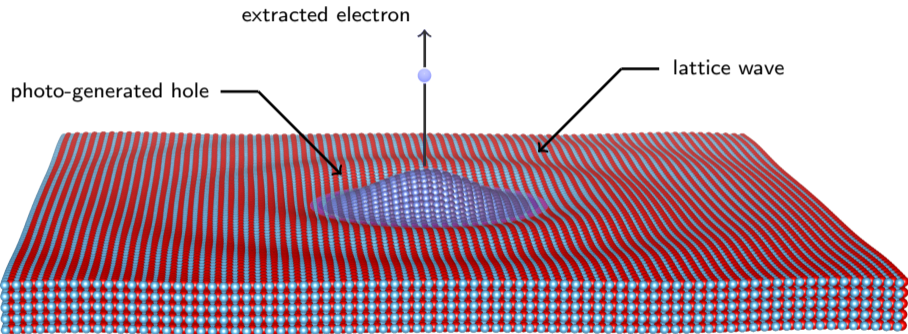


Riley et al, Nat. Commun. 9, 2305 (2018)

# Meaning of satellite bands

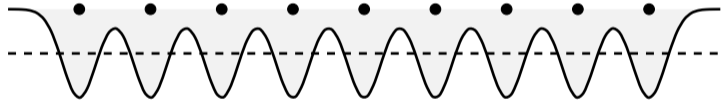
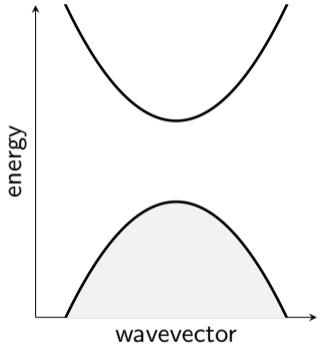


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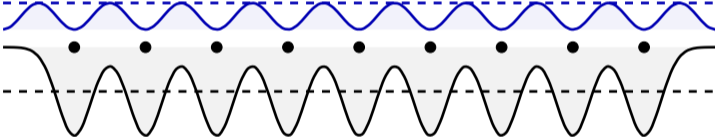
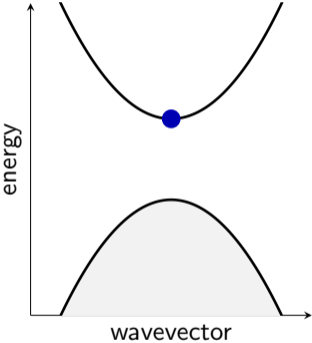


Satellites are shake-up excitations, the polaron is the QP peak

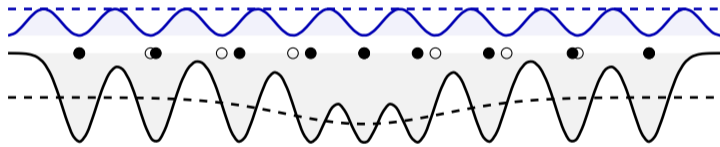
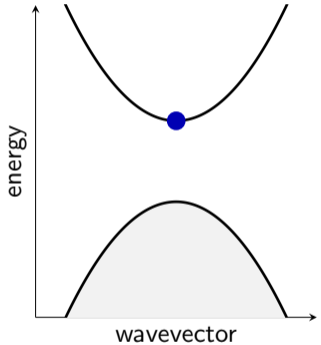
# Electron localization



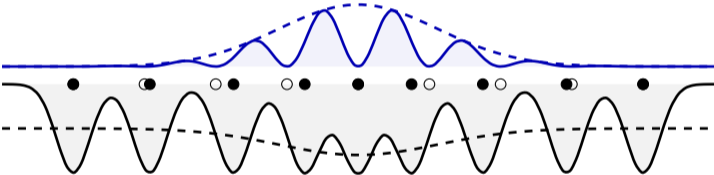
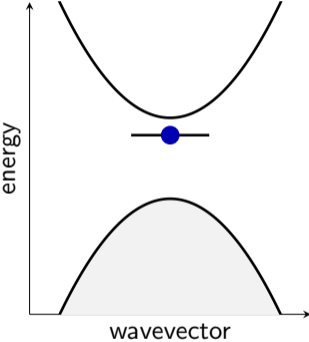
# Electron localization



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# Electron localization





## Ground state of the polaron in the Landau-Pekar model

$$E = \frac{\hbar^2}{2m^*} \int d\mathbf{r} |\nabla\psi|^2 + \frac{1}{2} \int d\mathbf{r} \mathbf{E} \cdot \mathbf{D}$$

Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

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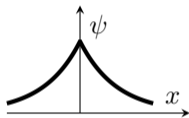
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$$-\frac{\hbar^2}{2m^*} \nabla^2 \psi(\mathbf{r}) - \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_0} - \frac{1}{\epsilon_\infty} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \psi(\mathbf{r}) = \epsilon \psi(\mathbf{r})$$

Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

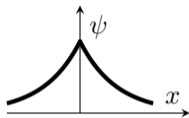
# Landau-Pekar equation

Simplest trial solution:  $\psi(\mathbf{r}) = \exp(-|\mathbf{r}|/r_p)$

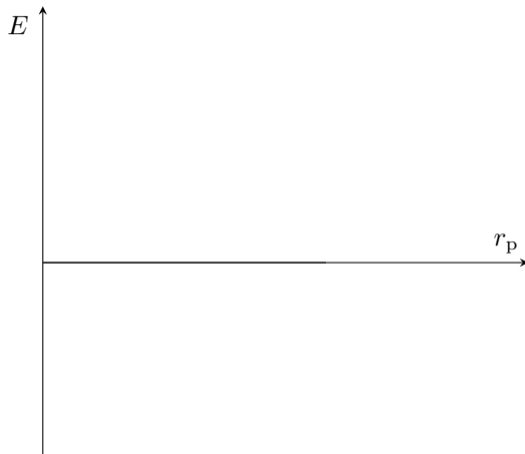


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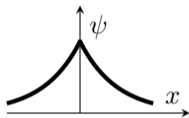


$E =$

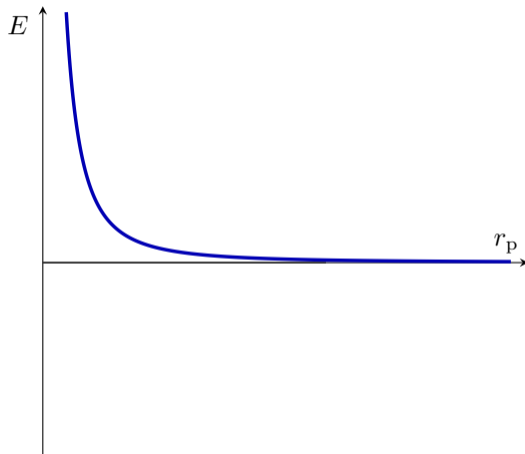


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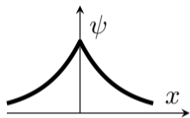
$$E = \frac{\hbar^2}{2m^*} \frac{1}{r_p^2}$$



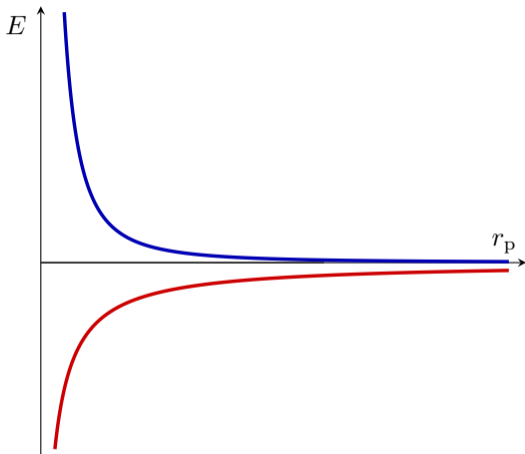


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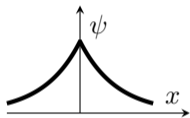


$$E = \frac{\hbar^2}{2m^*} \frac{1}{r_p^2} - \frac{5}{16} \left( \frac{1}{\epsilon_0} - \frac{1}{\epsilon_\infty} \right) \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_p}$$

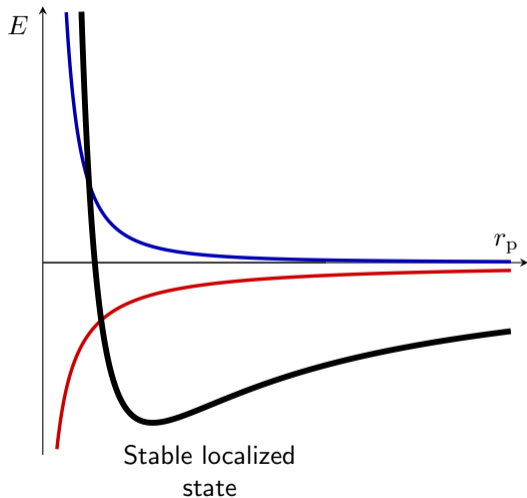


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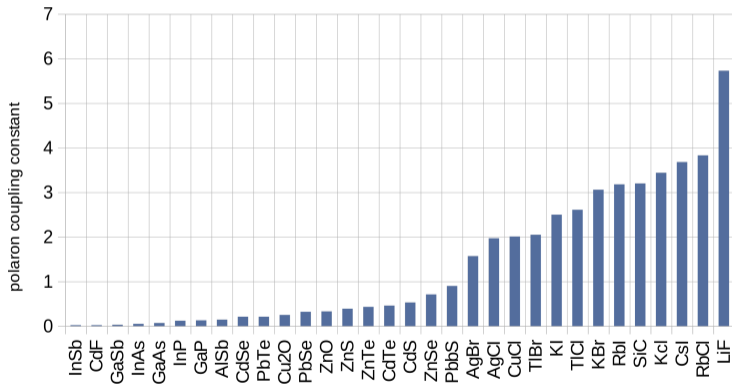


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# The polaron coupling constant

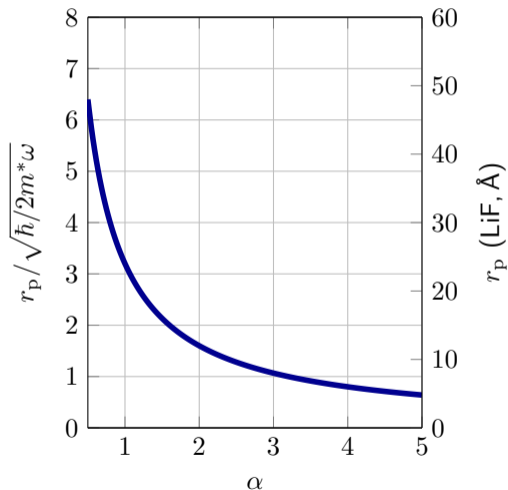
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar} \sqrt{\frac{m^*}{2\hbar\omega}} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right)$$



Data from Iadonisi, Riv. Nuovo Cim. 7, 1 (1984)

# Size of a polaron in the Landau-Pekar model

Radius: 
$$r_p = \frac{16}{5} \sqrt{\frac{\hbar}{2m^*\omega}} \frac{1}{\alpha}$$



# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state

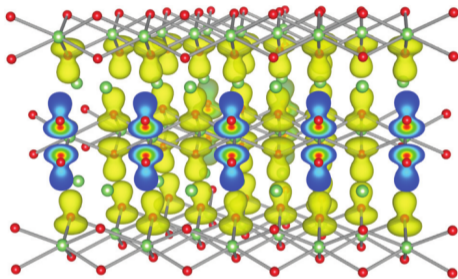
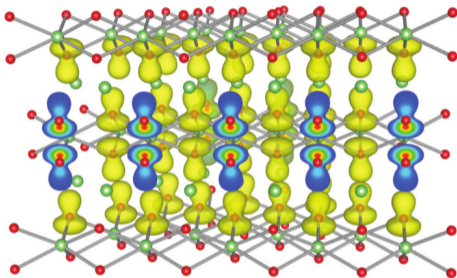


Figure from Feng et al, Phys. Rev. B 88, 184302 (2013)

# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state



Self-localization after ionic relaxation

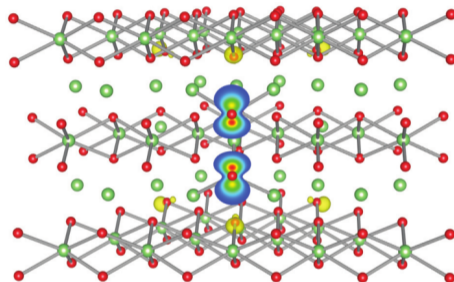
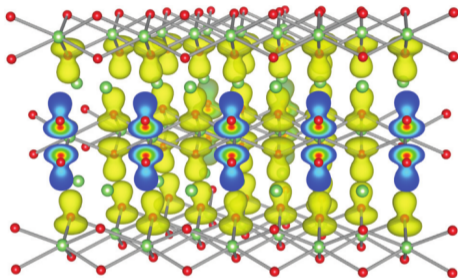


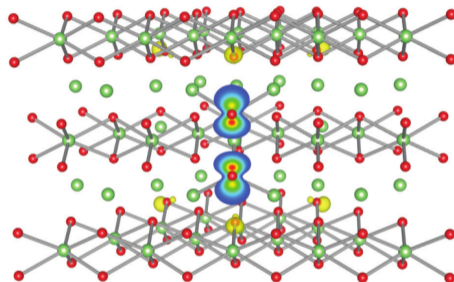
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# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state



Self-localization after ionic relaxation



Challenges { The energy and size of the localized state are sensitive to the XC functional  
Only small polarons accessible

Figure from Feng et al, Phys. Rev. B 88, 184302 (2013)

# Total energy in DFT

$$E = \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n]$$
$$+ \sum_{\kappa} \int d\mathbf{r} \frac{Z_{\kappa} n(\mathbf{r})}{|\mathbf{r} - \boldsymbol{\tau}_{\kappa}|} + \frac{1}{2} \sum_{\kappa \kappa'} \frac{Z_{\kappa} Z_{\kappa'}}{|\boldsymbol{\tau}_{\kappa} - \boldsymbol{\tau}_{\kappa'}|}$$



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Add one electron  $n(\mathbf{r}) \rightarrow n(\mathbf{r}) + |\psi(\mathbf{r})|^2$

$$\boldsymbol{\tau}_{\kappa} \rightarrow \boldsymbol{\tau}_{\kappa} + \mathbf{u}_{\kappa}$$

# Total energy in DFT

$$E =$$

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# Polarons in density-functional perturbation theory



Denny Sio

Total Energy functional of an extra electron or hole, without self-interaction

$$E = \int d\mathbf{r} \psi^* \hat{H}_{\text{KS}} \psi + \int d\mathbf{r} \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa\alpha}} |\psi|^2 u_{\kappa\alpha} + \frac{1}{2} C_{\kappa\alpha, \kappa'\alpha'} u_{\kappa\alpha} u_{\kappa'\alpha'}$$

# Polarons in density-functional perturbation theory



Denny Sio

Total Energy functional of an extra electron or hole, without self-interaction

$$E = \int d\mathbf{r} \psi^* \hat{H}_{\text{KS}} \psi + \int d\mathbf{r} \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa\alpha}} |\psi|^2 u_{\kappa\alpha} + \frac{1}{2} C_{\kappa\alpha, \kappa'\alpha'} u_{\kappa\alpha} u_{\kappa'\alpha'}$$

Variational minimization with respect to  $\psi$  and  $u_{\kappa\alpha}$

$$\begin{cases} \hat{H}_{\text{KS}} \psi + \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa\alpha}} \psi u_{\kappa\alpha} = \lambda \psi \\ u_{\kappa\alpha} = -(C)_{\kappa\alpha, \kappa'\alpha'}^{-1} \int d\mathbf{r} \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa'\alpha'}} |\psi|^2 \end{cases}$$

# Polarons in reciprocal space

$$\psi(\mathbf{r}) = \frac{1}{N_p} \sum_{n\mathbf{k}} A_{n\mathbf{k}} \psi_{n\mathbf{k}}(\mathbf{r})$$
$$u_{\kappa\alpha}(\mathbf{R}) = -\frac{2}{N_p} \sum_{\mathbf{q}\nu} B_{\mathbf{q}\nu}^* \left( \frac{\hbar}{2M_\kappa\omega_{\mathbf{q}\nu}} \right)^{1/2} e_{\kappa\alpha,\mathbf{q}\nu} e^{i\mathbf{q}\cdot\mathbf{R}}$$



# Polarons in reciprocal space

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$$\frac{2}{N_p} \sum_{\mathbf{q}m\nu} B_{\mathbf{q}\nu} g_{mn\nu}^*(\mathbf{k}, \mathbf{q}) A_{m\mathbf{k}+\mathbf{q}} = (\varepsilon_{n\mathbf{k}} - \varepsilon) A_{n\mathbf{k}}$$
$$B_{\mathbf{q}\nu} = \frac{1}{N_p} \sum_{m\mathbf{k}} A_{m\mathbf{k}+\mathbf{q}}^* \frac{g_{mn\nu}(\mathbf{k}, \mathbf{q})}{\hbar\omega_{\mathbf{q}\nu}} A_{n\mathbf{k}}$$

*Ab initio* polaron equations

# Electron polaron in LiF

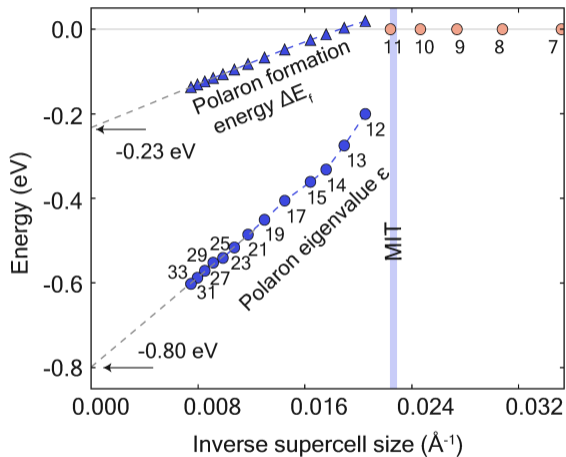


Figure from Sio et al, PRL 122, 246403 (2019)

# Electron polaron in LiF

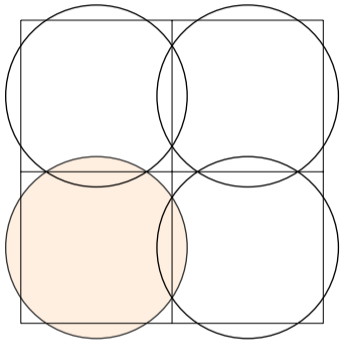
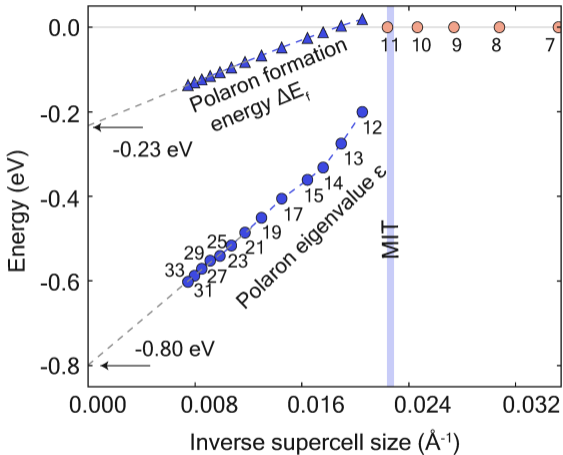


Figure from Sio et al, PRL 122, 246403 (2019)

# Electron polaron in LiF

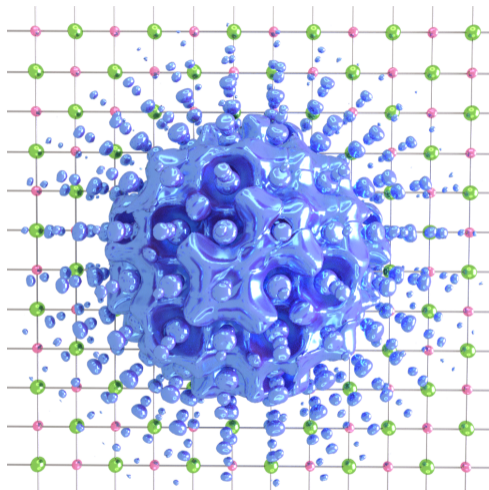
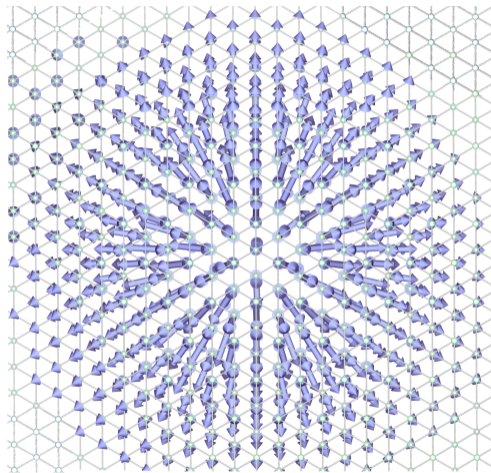


Figure from Sio et al, PRL 122, 246403 (2019)

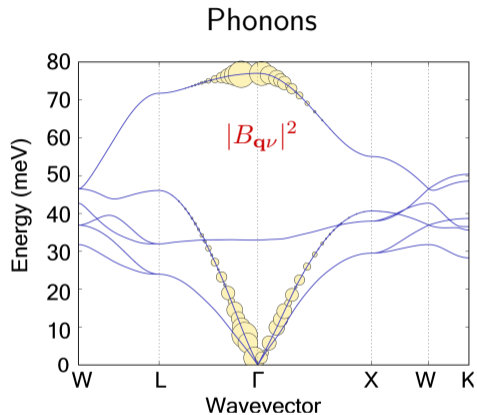
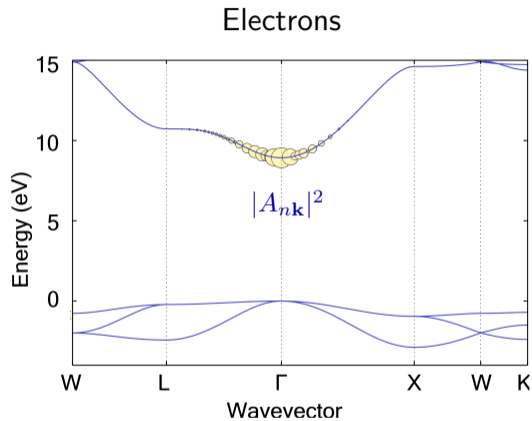
# Electron polaron in LiF



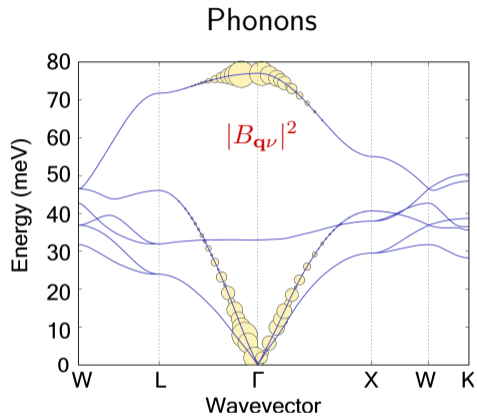
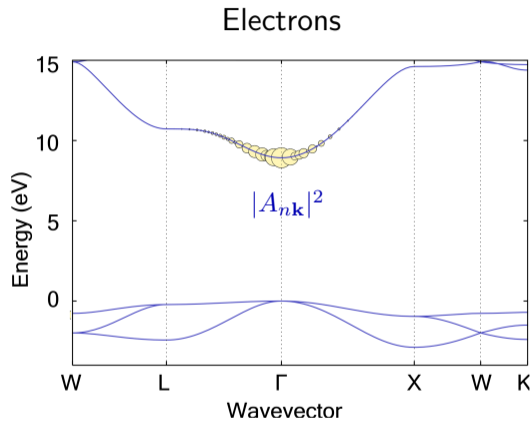
fluorine displacements

Figure from Sio et al, PRL 122, 246403 (2019)

# Example: Electron polaron in Lithium Fluoride



# Example: Electron polaron in Lithium Fluoride



Fröhlich polaron

# Hole polaron in LiF

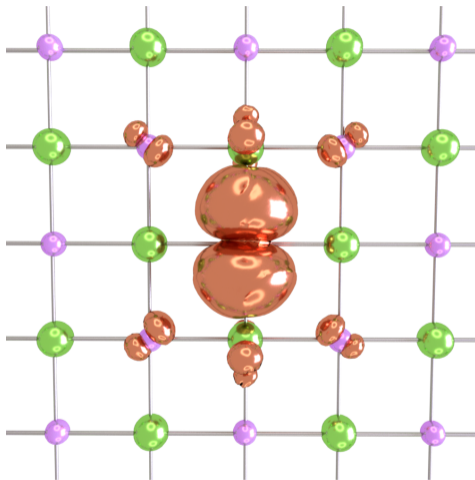
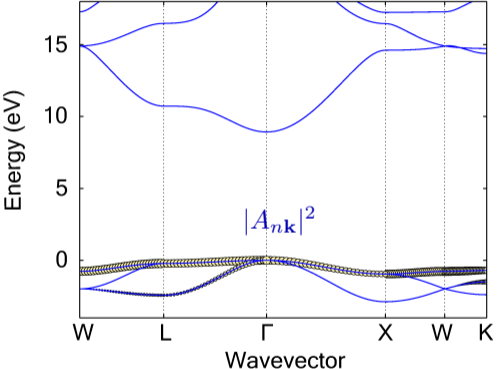


Figure from Sio et al, PRB 99, 235139 (2019)

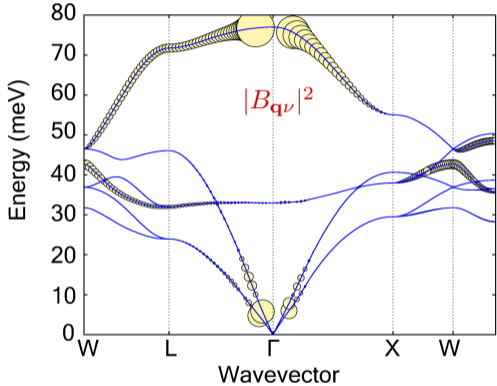


# Example: Hole polaron in LiF

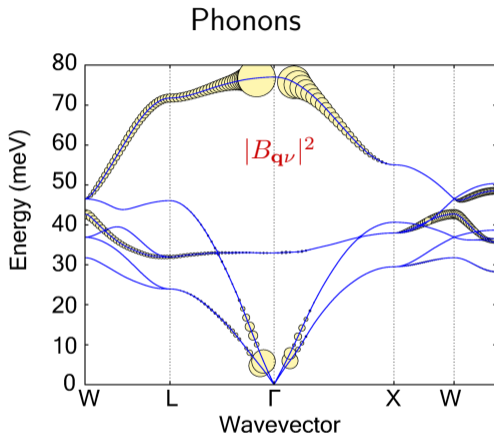
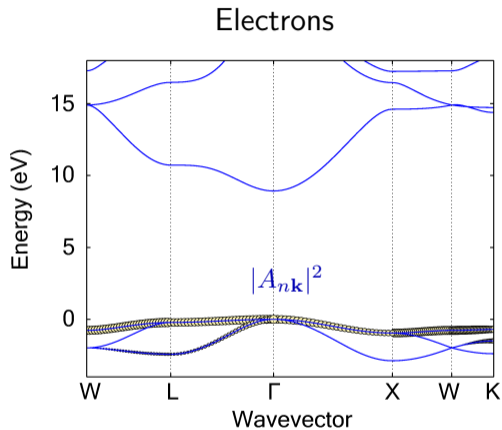
## Electrons



## Phonons

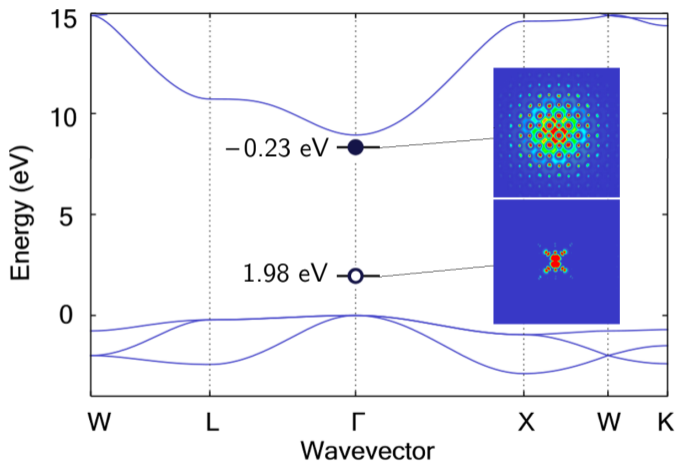


# Example: Hole polaron in LiF



Holstein polaron

# Quasiparticle energies of polarons in LiF



Shown are QP energies, eg  $E_{N+1} - E_N$

## Effect of self-interaction: A lesson from the Landau-Pekar model

$$\hat{H}_{\text{LP}} = -\frac{\hbar^2}{2m^*} \nabla^2 - \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$

## Effect of self-interaction: A lesson from the Landau-Pekar model

$$\hat{H}_{\text{LP}} = -\frac{\hbar^2}{2m^*} \nabla^2 - \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} + \boxed{\frac{e^2}{4\pi\epsilon_0} \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}}$$

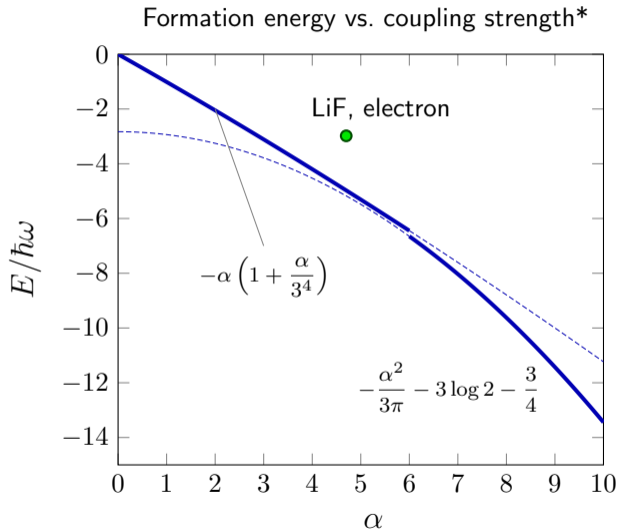
## Effect of self-interaction: A lesson from the Landau-Pekar model

$$\hat{H}_{\text{LP}} = -\frac{\hbar^2}{2m^*}\nabla^2 - \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} + \boxed{\frac{e^2}{4\pi\epsilon_0} \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}}$$

$$\hat{H}_{\text{LP}} = -\frac{\hbar^2}{2m^*}\nabla^2 - \frac{e^2}{4\pi\epsilon_0} \underbrace{\left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} - 1 \right)}_{[-1,0]} \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$

- Hartree self-interaction suppresses localization
- Hybrid functionals partly cancel self-interaction

# Feynman's polaron

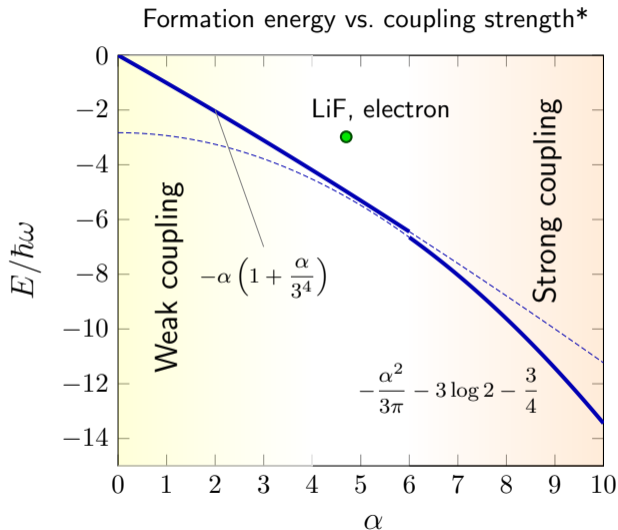


Similar to DMC results by  
Mishchenko et al,  
Phys. Rev. B 62, 6317 (2000)

\*Valid only for Fröhlich model

From: Feynman and Hibbs, p. 318

# Feynman's polaron



Similar to DMC results by  
Mishchenko et al,  
Phys. Rev. B 62, 6317 (2000)

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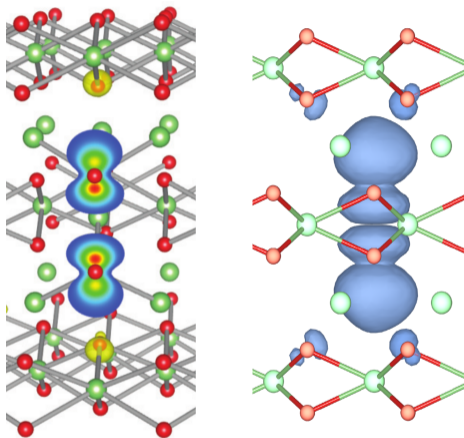
From: Feynman and Hibbs, p. 318



- Many-body approach provides spectral function of polarons, but no wavefunction
- DFPT approach provides wavefunction of polaron, but no spectral function
- Progress in the study of polarons in real materials will likely require a combination of these approaches

- Franchini et al, Nat. Rev. Mater. 2021 [\[link\]](#)
- Devreese et al, Rep. Prog. Phys. 72, 066501 (2009) [\[link\]](#)
- Devreese, arXiv:1611.06122 (2020) [\[link\]](#)
- FG, Rev. Mod. Phys. 89, 015003 (2017) [\[link\]](#)
- Sio et al, Phys. Rev. Lett. 122, 246403 (2019) [\[link\]](#)
- Verdi et al, Nat. Commun. 8, 15769 (2017) [\[link\]](#)
- Nery et al, Phys. Rev. B 97 (2018) [\[link\]](#)
- Lee et al, arXiv:2011.03620 (2020) [\[link\]](#)

# Perturbation approach vs. hybrid DFT: $\text{Li}_2\text{O}_2$



Left figure from Feng et al, PRB 88, 184302 (2013); Right figure from Sio et al, PRL 122, 246403 (2019)